

### 10.3 Hypothesis Tests For A Population Mean

The hypothesis test for a population mean parallels that of a population proportion with a few small differences. The following conditions must be satisfied in order for our hypothesis testing techniques to valid

- The sample is obtained using simple random sampling techniques
- The sample must not contain outliers and the population from which the sample is drawn is normally distributed or the sample size is large ( $n \geq 30$ ).

In addition, the test statistic will be t-values and critical values will come from Student's t-distribution with  $n-1$  degrees of freedom. The test statistic will be calculated using the following formula:

$$t = \frac{\bar{x} - \mu_o}{\frac{s}{\sqrt{n}}}$$

**Example 1:** The student affairs office at a college was asked the following question. “How far away does the average community college student commute to college every day?”. The director answered “No more than 9 miles.” The college president was not convinced that the response was accurate. As a result, she conducted her own study. She selected a random sample of 50 commuter students and found the mean distance travelled to be 10.22 miles. Test the hypothesis stated above at a significance level of  $\alpha = 0.05$  using a standard deviation from previous studies of 5 miles.

**Example 2:** A researcher reports that the average salary of assistant professors is more than \$42,000. A sample of 30 assistant professors has a mean salary of \$43,260. At a level of significance of  $\alpha = 0.01$ , test the claim that assistant professors earn more than \$42,000 per year. The standard deviation of the population is \$5230.

If sample size is small ( $n < 30$ ) then we must verify that the sample comes from a population that is approximately normally distributed with no outliers. A **normal probability plot** and a **box plot** will be needed for this purpose.

Also, when conducting a statistical hypothesis test using sample data, one may find that although there is a statistical difference between two values however there is no practical difference between values.

**Example 3:** Use a p-value approach. According to the American Community Survey, the average travel time to work in Collin County Texas was 27.3 minutes. Shortly after, the traffic lights in the county were recalibrated in an attempt to reduce travel time. To determine if the recalibration was effective the department of transportation obtains a random sample of 2500 commuters and finds the mean travel time now is 27.0 minutes with a standard deviation of 8.5 minutes. Using a level of significance of 0.05 does this suggest that the travel time has decreased?